Weston ENGINEERING NOTES

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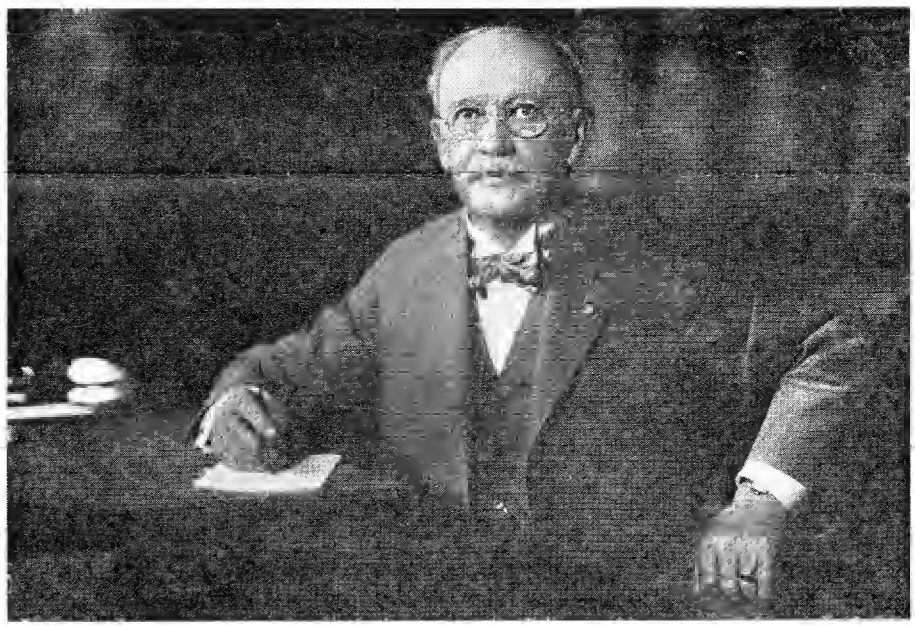
ELECTRICAL RESISTANCE ALLOYS

ELECTRICAL Resistance Alloys, as is well known, have relatively high resistivities, and preferably are little affected by changes in temperature. They are used in electric circuits to limit or control the current flowing, or for purposes of heating. Two of the best and oldest of these alloys are manganin and constantan. They have high resistivities and the effect of temperature upon their resistivities is practically negligible.

Constantan is also a very stable alloy, but it has a high thermo-electromotive force against copper, and for this reason can not be used with accuracy in precision instruments intended for measurement in the millivolt range. Because of its high thermo emf against copper and some other metals, constantan has been standardized as one of the elements in a thermocouple for use in temperature measurements.



Dr. Edward Weston (1850-1936), founder of the Weston Electrical Instrument Corporation, discovered manganese-copper-nickel and copper-nickel negligible temperature coefficient alloys in 1884.

Manganin, because of its remarkable constancy under all conditions, is most suitable for use as series or adjusting resistors in electrical measuring instruments, and in Wheatstone bridges and other precision electric devices including resistance standards. In fact, the National Bureau of Standards employs manganin resistors to maintain the standard ohm, and together with the Weston Normal Cell, to maintain the standard ampere.

These two alloys are so well known and widely used that one is apt to take them for granted without giving their origin much thought, despite the fact that their discovery was epoch making. For this reason it was thought that a short history of their development might prove of interest to many readers.

The impression seems to persist in the minds of some scientists and technicians that the negligible temperature coefficient resistance alloys

In This Issue

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were discovered in Germany. The fact is that they were discovered after extensive research in the year 1884 by Dr. Edward Weston in Newark, New Jersey.

Patents Granted

In the year 1885 Dr. Weston applied for U. S. Patents on manganese-copper-nickel, and manganese-copper alloys, and in 1888 was granted Reissue Patents No. 10944 and No. 10945, respectively.

These patents include claims which, as a result of an entirely new art, are among the broadest ever granted by the Patent office and it may be of interest to quote them here.

Patent Re No. 10944:

"A metallic electrical conductor the electrical resistance of which diminishes with increase of temperature.

"An electrical conductor consisting of an alloy of copper or its equivalent, manganese, and nickel."

Patent Re No. 10945:

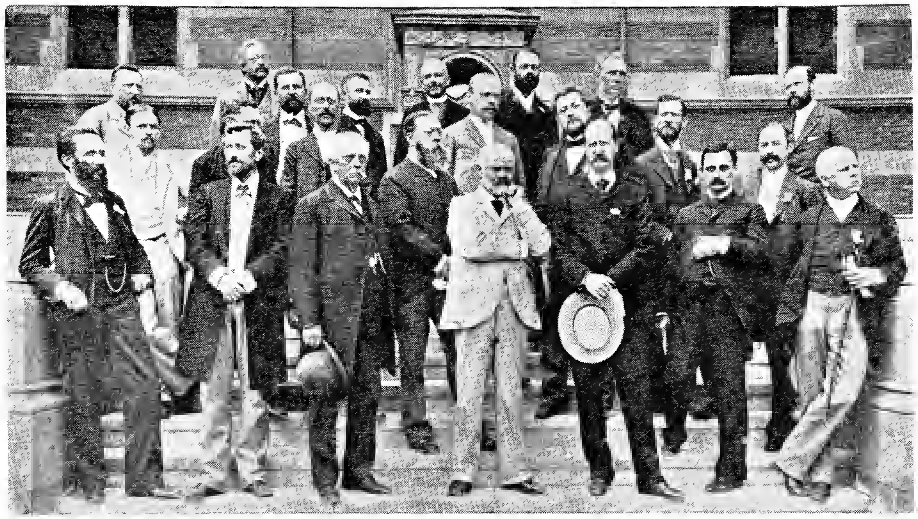
"An electrical conductor consisting of a material the electrical resistance of which is substantially constant under varying temperatures.

"In an electrical apparatus, a conductor composed of an alloy containing manganese and copper or its equivalent."

The discovery of these alloys was a very startling one at the time, since not only did Dr. Weston find an alloy which had a negligible temperature coefficient of resistance, but some having a negative temperature coefficient. Prior to this discovery, it was considered by physicists that the principal distinguishing characteristics of metallic substances, and non-metallic substances such as carbon, were that the resistivity of metallic substances increased as the temperature increased whereas that of non-metallic substances decreased with an increase in temperature.

As will be referred to later in more detail, Dr. Weston also investigated the copper-nickel alloys and found a composition which had a negligible temperature coefficient, which later became known as constantan.

The original designations Dr. Weston gave to the two alloys were No. 2 Alloy for the copper-nickel



Official Delegates to the International Electrical Congress held at Chicago in 1893.

BUDDE .. SIEMENS LEDUC NICHOLS WENNMAN THURY
SCHRADER DE LA TOUANNE PALAZ HOSPITALIER CARHART
LUMMER VOIT VIOLLE PREECE S. P. THOMPSON
FERRARIS AYRTON VON HELMHOLTZ MASCART ROWLAND E. THOMSON MENDENHALL

alloy subsequently known as constantan, and No. 3 Alloy for the alloy subsequently known as manganin, and these numbers are still used familiarly in the Weston organization.

The reason it was generally assumed that these alloys were discovered in Germany was that the German Physikalische Technische Reichsanstalt, the German Bureau of Standards, saw Dr. Weston's patents and started their own investigations on the material. After corroborating his results, they made a study of the exact proportions of copper, nickel, and manganese to best suit their purpose, which they standardized and named the material manganin. The German Reichsanstalt was the only important standardizing body in the world at that time as the Bureau of Standards of the United States and that of England were not yet in existence.

Another reason for the incorrect assumption as to the origin of the low temperature coefficient alloys was that in arranging for the commercial production of the materials, Dr. Weston could not find mills in this country able or willing to undertake the work of producing them. However, the well-known German firm, Isabellenhütte, undertook to do the work and for a number of years the Weston Company imported the materials until the Weston plant was equipped with proper facilities for producing them commercially.

Recognition by Great Scientists

Even so well informed a man as Lord Kelvin, the great English scientist, incorrectly assumed that these alloys were discovered by the German Reichsanstalt until he was corrected by Prof. von Helmholtz of the German Reichsanstalt. This occurred during a meeting of the British Association held in London in 1892. The meeting was for the purpose of considering the desirability of establishing a National Physical Laboratory for England similar to the German Reichsanstalt which had been in use for about two years. The discussion was by Lord Kelvin and Prof. Ayrton of England and Prof. von Helmholtz of Germany. The report of the discussion which ensued was published in the *Electrician* (London) of September 2, 1892, Volume 29, Pages 495-496, which seems pertinent and of sufficient interest to quote in full as follows:

Lord Kelvin—"The grand success of the Physikalische Reichsanstalt may be judged to some extent here by the record put before us by Prof. von Helmholtz. Such a proved success may be followed by a country like England with very great profit indeed. One thing Prof. von Helmholtz did not mention was the discovery in the Anstalt of a metal whose temperature co-

efficient in respect of electrical resistance is practically nil; that is to say, a metal whose electrical resistance does not change with temperature. This is just the thing that we have been wanting for twenty or thirty years. It is of the greatest importance in scientific experiments, and also in connection with the measuring instruments of practical electric lighting, to have a metal whose electrical resistance does not vary with temperature; and after what has been done, what is now wanted is to find a metal of good quality and substance whose resistance will diminish as temperature is increased. We want something to produce the opposite effect to that with which we are familiar. The resistance of carbon diminishes as temperature increases; but its behavior is not very constant. Until within the last year or so nothing different was known of metals from the fact that the elevation of temperature had the effect of increasing resistance. The Physikalische Anstalt had not been in existence two years before this valuable metal was discovered."

Prof. von Helmholtz—"The discovery of a metal whose resistance diminished with temperature was made by an American engineer."

Prof. Ayrton—"By an Englishman—Weston."

Lord Kelvin—"That serves but to intensify the position I wished to take, whether the discovery was made by an Anglo-American, an American Englishman, or an Englishman in America. It is not gratifying to national pride to know that these discoveries were not made in this country (England)."

An International Electrical Congress was held at Chicago in 1893 in connection with the World's Columbian Exposition and a photograph of the official delegates is shown on the opposite page. Among these delegates, many of whom have since become historically famous, are several who specifically acknowledged Dr. Weston's discovery of copper-nickel and copper-nickel-manganese alloys. In a paper published in the *Proceedings* of the Congress, Page 172, Dr. Stephen Lindeck, a member of the German Reichsanstalt, made the following statement:

"On the whole, our experience has led us to the conclusion that for standards such alloys do best which beside copper and nickel, also contain manganese. Some years ago Mr. Weston of Newark, New Jersey, discovered that alloys containing manganese possess a very small temperature coefficient, and that it is even possible to obtain metals with a negative temperature coefficient in this way. . . . After hearing of Mr. Weston's observations, the further investigation of manganese alloys was taken up at the Reichsanstalt, and we obtained very interesting results."

Feusner and Lindeck, both members of the German Reichsanstalt, and who first utilized manganin for their resistance standards, in a paper entitled "Metal Alloys for Electrical Resistances," published in *Zeitschrift für Instrumentenkunde*, 1889, Volume 9, Page 235, refer to Dr. Weston's original work on the manganese alloys having negligible and negative temperature coefficients.

The Scientific Instrument Catalogue of the German Educational Exhibit at St. Louis in 1904 stated:

"We will specially mention here those instruments which lie at the foundation of direct current measurement and which indicate the work of the Physikalische Reichsanstalt in this field.

"Edward Weston of Newark, New Jersey, has given a special impetus to work in this direction. A patent taken out by him has led the Reichsanstalt (Feusner and Lindeck) to the investigation of alloys containing manganese, resulting in the adoption of manganin for standard resistors."

Dr. Wilhelm Jaeger, a member of the German Reichsanstalt from its inception, in his book entitled *Electrische Messtechnik 1917*, on Page 261 in referring to alloys for use in resistance standards, states: "Of all alloys suitable for this purpose the best is that proposed by Weston of Newark, Manganin. . . ."

The German magazine *Die Messtechnik* of June 24, 1927, in reference to some of Dr. Weston's work states: "Weston is to be thanked further (in addition to other contributions) for alloys with negligible or negative temperature coefficients, as are now used in the instrument art and also in the construction of precision resistors."

The numerous references just given show that the German scientists laid no claim to the discovery of the negligible and negative temperature coefficient alloys but gave the credit to Dr. Weston.

German Silver Proves Inadequate

Prior to the year 1884, when Dr. Weston discovered the manganese alloys, the alloys available for use in standards or instruments were German silver, designated by the Germans as neusilber, and platinum silver. At a later date, modifications of German silver were developed, such as platenoid and nickelin which was German silver having a nickel content of 18 per

Electrical and Thermal Characteristics of Manganin and Constantan

	Manganin	Constantan
Electrical resistivity; microhms per cm³	48	49
Electricol resistivity; ohms per circ. mil. ft.	290	294
Temperature coefficient of resistance per deg. C.	*±0.00001	−0.00001
Thermo emf against copper, microvolts per deg. C.	1.7	†43
Thermal conductivity; watts per deg. C. per cm³		
at 20 C.	0.220	0.226
at 100 C.	0.264	0.266
Thermol capacity; joules per gram per deg. C.		
at 20 C.	0.408	0.410
at 100 C.	0.420	0.427

NOTES:

* *The temperature coefficient of manganin is zero at some temperature in the range from 25 C. to 55 C., depending upon the actual composition of the alloy and its manufacturing process. Below and above this temperature the coefficient is positive and negative respectively, having a maximum effective value within the working temperature range of about ±0.00001.*

† *The thermo emf of constantan against copper varies somewhat with the temperature range and the actual temperature of the junction. For very accurate values, tables should be consulted.*

cent. All of these alloys were considerably affected by changes in temperature.

German silver usually consisted of about 14.6 per cent nickel, 25.4 per cent zinc and 60 per cent copper. It had a resistivity of about 13 times that of copper, and a temperature coefficient of about 0.00044 ohm per ohm per deg. cent. The maximum nickel content in German silver available at that time was 18 per cent.

In the year 1883, after considerable research, Dr. Weston was finally able to produce a workable copper-nickel-zinc alloy of the German silver type having a nickel content of 30 per cent and above. He proved that the resistivity was correspondingly increased and its temperature coefficient reduced as a result of increased nickel content. This material had a resistivity of 28 times that of copper and a temperature coefficient 0.00033 per deg. cent.

Dr. Weston found, however, that German silver was not stable. In fact, when used in the laboratory and exposed to the air, it soon became brittle and finally would break up into powder. He attributed this to the zinc content, and thereupon experimented on zinc-free copper-nickel alloys and found that he could produce alloys which had a very high resistivity and negligible temperature coefficient. The alloy having a 40 per cent nickel content, as stated previously, he designated No. 2 Alloy, which was subsequently named constantan, probably by the German Reichsanstalt, or by Isabellenhütte who manufactured the alloy commercially.

These copper-nickel alloys were excellent except that they had a very large thermo-electromotive force against copper. Dr. Weston thereupon continued a very intensive research using alloys of various metals, including the noble metals. He made and tested upward of 400 different alloys and finally discovered the manganese copper, and manganese-nickel-copper alloys having a negligible and even a negative temperature coefficient. These he patented as noted above, and designated the latter as No. 3 Alloy, which, however, was named manganin by the German Reichsanstalt.

Manganin is normally composed of 84 per cent copper, 12 per cent manganese and 4 per cent nickel. Constantan consists of 40 per cent nickel and 60 per cent copper.

The table shown on page 3 lists the most useful electrical and thermal constants of the two alloys.

E. N.—No. 35 —*W. N. Goodwin, Jr.*

THE EFFECT OF HARMONICS ON A-C AMMETER AND VOLTMETER INDICATIONS

ALTERNATING current instruments of the iron vane repulsion type, electrodynamometer type and thermocouple type, are used to measure the rms value of alternating currents and voltages. It is well known that these instruments will indicate correctly if the frequency of the fundamental and any harmonics present in the quantity being measured is within the frequency rating of the instrument. If, however, the frequency of the harmonics is beyond the frequency range of the instrument, a significant error may exist depending upon the frequency and magnitude of the harmonics present. This error can be computed for the instruments of the above types providing they do not have a saturating iron circuit.

The rms value of a non-sinusoidal current is equal to the square root of the sum of the squares of the fundamental and harmonics. This may be expressed as

$$I=\sqrt{I_f^2+I_2^2+I_3^2+\ldots+I_n^2} \qquad (1)$$

where I = rms value of current
I_f = rms value of the fundamental
I_2, I_3, I_n = rms values of the harmonics.

An a-c instrument will indicate this summation correctly only when it responds properly to the fundamental and each harmonic. If, for example, the indications of an ammeter are correct at the fundamental frequency and tend to decrease at the frequency of the third and fifth harmonic, the summation will be smaller and the value indicated will be lower than the true rms value. If the actual response to each harmonic is known, then the indicated current

$$I_i=\sqrt{(P_1I_f)^2+(P_2I_2)^2+(P_3I_3)^2+\ldots+(P_nI_n)^2} \qquad (2)$$

where P_1, P_2, P_3 and P_n are the ratios of the indicated values to the true values. The ratio of the indicated current to true current is therefore

$$\frac{I_i}{I}=\sqrt{\frac{(P_1I_f)^2+(P_2I_2)^2+(P_3I_3)^2+\ldots\ldots+(P_nI_n)^2}{I_f^2+I_2^2+I_3^2+\ldots\ldots+I_n^2}} \qquad (3)$$

and the error in per cent of reading is

$$Error=100\left(\frac{I_i}{I}-1\right) \qquad (4)$$

Since voltage and current waves can be analyzed and the percentage of harmonics determined with respect to the fundamental, the ratio in general can be expressed as

$$\frac{I_i}{I}\text{ or }\frac{E_i}{E}=\sqrt{\frac{(P_1H_f)^2+(P_2H_2)^2+(P_3H_3)^2+..+(P_nH_n)^2}{H_f^2+H_2^2+H_3^2+..+H_n^2}} \qquad (5)$$

The error is then

$$100\left[\sqrt{\frac{(P_1H_f)^2+(P_2H_2)^2+(P_3H_3)^2+\ldots+(P_nH_n)^2}{H_f^2+H_2^2+H_3^2+\ldots+H_n^2}}-1\right] \qquad (6)$$

where H_f = fundamental
H_2, H_3, H_n = value of the harmonic expressed as a fraction of the amplitude of the fundamental.

The error can therefore be computed if a harmonic analysis of the wave in question is available, whereby each harmonic is evaluated as a percentage of the

fundamental, and if the error of the instrument is known for each of these frequencies. The frequency errors of ammeters calibrated at 60 cycles are usually small, being in the order of 0.5 to 1 per cent at 500 cycles and in direct proportion for other frequencies up to 2000 or 3000 cycles. The frequency errors of uncompensated voltmeters calibrated at 60 cycles may be as high as 15 or 20 per cent at 500 cycles and will vary approximately as the frequency squared. The frequency errors of voltmeters and ammeters can be determined by test or the voltmeter frequency error can be computed if the inductance and resistance are known.

Typical Example

Let us assume a voltmeter is being used to measure the rms voltage of a 60-cycle wave having a 33.3 per cent third harmonic, a 20 per cent fifth harmonic and a 14.3 per cent seventh harmonic. This wave is shown in Figure 1.

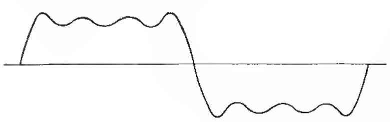


Figure 1—Voltage wave consisting of 33.3% third harmonic, 20% fifth harmonic and 14.3% seventh harmonic.

The voltmeter has been calibrated on a 60-cycle sine wave source and has errors at other frequencies (frequency error) as shown in Figure 2. The frequency errors at the harmonic frequencies of 180, 300 and 420 cycles as determined from the curve are −1.8, −5.4 and −10.8 per cent of reading, respectively. The value of P is computed from this error since

$$P = \frac{100 + \mathit{Frequency\ Error}}{100} \qquad (7)$$

The values of P and H can be tabulated as follows:

Harmonic	Frequency	Frequency Error	P	H
Fund.	60	0	1.000	1.000
3rd	180	−1.8	0.982	.333
5th	300	−5.4	0.946	.200
7th	420	−10.8	0.892	.143

Error =

$$100\left[\sqrt{\frac{(1\times 1)^2+(.982\times .333)^2+(.946\times .2)^2+(.892\times .143)^2}{1^2+\overline{.333}^2+\overline{.20}^2+\overline{.143}^2}}-1\right]$$

$= 100\ (.99472\text{-}1) = -0.528$ *per cent.*

Simplified Equation

By using the approximations $\sqrt{xy} = \frac{x+y}{2}$ when the values of x and y are nearly equal and $(1 \pm z)^2 = 1 \pm 2z$ when z is a small quantity, equation (6) can be reduced to a simplified form sufficiently accurate for normal use. It can be shown that the approximate error is

$$\mathit{Error} = \frac{e_2H_2^2 + e_3H_3^2 + \ldots + e_nH_n^2}{H_f^2 + H_2^2 + H_3^2 + \ldots + H_n^2} \qquad (8)$$

where e_2, e_3, e_n = frequency error in per cent of reading. By using this equation for the example shown previously,

$$\mathit{Error} = \frac{-1.8\times\overline{.333}^2 - 5.4\times\overline{.20}^2 - 10.8\times\overline{.143}^2}{1^2 + \overline{.333}^2 + \overline{.20}^2 + \overline{.143}^2}$$

$= -0.543\%$

This error is slightly greater than that obtained using formula (6) but since the difference is so small (.015 per cent), equation (8) can be used for most applications.

Error for One Harmonic

If only one harmonic is present in considerable magnitude, the error from the approximate equation is

$$\mathit{Error} = \frac{eH^2}{1+H^2} \qquad (9)$$

where e = frequency error at the harmonic frequency in per cent of reading.
H = value of the harmonic expressed as a fraction of the amplitude of the fundamental.

This equation is very useful since it shows that the error will vary directly as the frequency error of the instrument and approximately as the square of the harmonic amplitude.

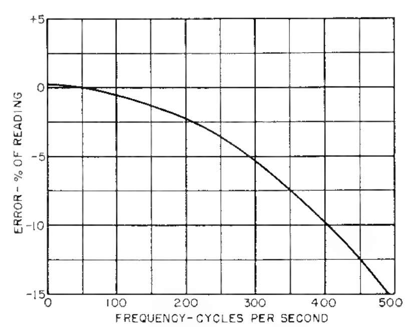


Figure 2—Voltmeter Frequency Error.

The presence of harmonics in the quantity being measured does not mean that the instrument will have a large error. The typical example shows that a voltmeter used to measure a 60-cycle voltage having as much as 33.3 per cent third harmonic, 20 per cent fifth harmonic and 14.3 per cent seventh harmonic has an error of less than 0.6 per cent even when the frequency error of the instrument is 10 per cent at the seventh harmonic (420 cycles). Ammeters which have much smaller frequency errors will be even less affected.

Caution should be exercised when using voltmeters on high power frequencies having harmonics present

in considerable magnitude. Voltmeters adjusted for use on frequencies of 360 to 440 cycles may have very large frequency errors at the third, fifth and seventh harmonics. For this application, voltmeters compensated for a frequency span of 25-1000 cycles or higher should be used since frequency errors at the harmonic frequencies are usually small and the harmonic errors will be reduced to a negligible value. The proper instrument to use when harmonics are present is one which does not have large frequency errors at the harmonic frequencies.

E. N.—No. 36 —*R. F. Estoppey*

WHEATSTONE BRIDGE—LIST OF CIRCUIT EQUATIONS

IN AN article in the WESTON ENGINEERING NOTES, Volume 1, Number 1, equations were given for determining the galvanometer current in a bridge network, and also for the bridge resistance. These have been found useful by readers, and requests have been received for equations for currents in all branches of the bridge under various conditions.

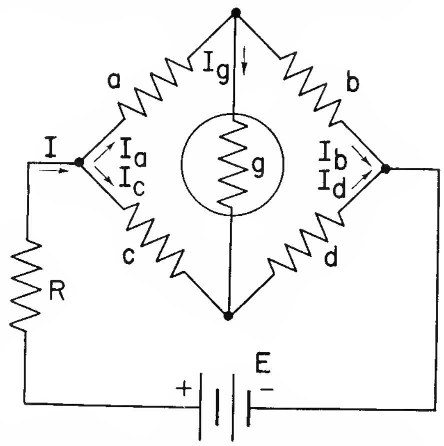
In the tabulated list which follows, equations for the currents in all the branches of the bridge circuit are given, both in terms of the battery current, and in terms of the battery voltage.

In developing these equations many years ago, as a check on the few to be found in the literature, the author discovered that if two constants, designated as A and B, were first computed, the equations expressed in terms of these constants became much simpler. The equations are therefore given in two forms, the first one simplified by the use of the constants A and B, and the second complete in itself for independent computation. If only one current is to be computed, it will be found simpler to use the complete self-contained form of equation.

Equations are given also for the resistance of the bridge looking from the battery, and for the resistance looking out from the galvanometer, which is its shunt or damping circuit resistance. Equations for bridge sensitivity are given and also the conditions for the maximum wattage in the galvanometer, to determine the optimum bridge and galvanometer sensitivity.

In the sketch the arrows show the direction of currents for positive values as derived from the equations. If a negative value results in applying the equations for numerical computations, the direction of the current is opposite to that of the arrow.

The symbols R, a, b, c, and d are values of resistances, and I, I_a, I_b, I_c, and I_d are values of currents.



The Conventional Wheatstone Bridge Diagram.

Alternating Current Bridge

The equations given may be used equally well for alternating current operation, by remembering that in this case, the symbols a, b, c, etc., represent impedances expressed as complex quantities.

For example, a becomes $r_a + jx_a$, where r_a is the resistance, x_a is the reactance and $j = \sqrt{-1}$. For inductive reactance, x_a is positive, whereas for capacitive reactance x_a is negative. A similar treatment applies to all of the other quantities. It must be remembered that in the equation for a bridge balance, $bc = ad$, for alternating current use, it is necessary that the real and imaginary quantities, that is the in-phase and quadrature components on one side of the equation must be equal to the in-phase and quadrature components respectively on the other side. The values of the currents obtained then give the component of the current in phase with the total bridge current, or applied emf according to the equation used, and the component in quadrature with them.

Values of Constants *A* and *B*

$$A = b(a+c) + g(a+b) \qquad (1)$$

$$B = a(b+d) + g(a+b) \qquad (2)$$

The complete equations are designated by the letter (C) and those simplified by using the constants A and B are designated by the letter (S).

Equations for Currents in Terms of the Battery Voltage E, With Series Resistance R

$$I_g = \frac{E(A-B)}{\frac{R}{a+b}[B(a+c)+A(b+d)]+Bc+Ad} \qquad \text{(S)} \quad (3)$$

$$I_g = \frac{E(bc-ad)}{R[(a+c)(b+d)+g(a+b+c+d)]+ab(c+d)+dc(a+b)+g(a+b)(c+d)} \qquad \text{(C)} \quad (4)$$

$$I_a = \frac{\frac{E}{b}[(A-B)(g+b)+Ad]}{\frac{R}{a+b}[B(a+c)+A(b+d)]+Bc+Ad} \qquad \text{(S)} \quad (5)$$

$$I_a = \frac{E[c(b+d)+g(c+d)]}{R[(a+c)(b+d)+g(a+b+c+d)]+ab(c+d)+dc(a+b)+g(a+b)(c+d)} \qquad \text{(C)} \quad (6)$$

$$I_b = \frac{\frac{E}{b}[(A-B)g+Ad]}{\frac{R}{a+b}[B(a+c)+A(b+d)]+Bc+Ad} \quad \text{(S)} \quad (7)$$

$$I_b = \frac{E[d(a+c)+g(c+d)]}{R[(a+c)(b+d)+g(a+b+c+d)]+ab(c+d)+dc(a+b)+g(a+b)(c+d)} \quad \text{(C)} \quad (8)$$

$$I_c = \frac{EB}{\frac{R}{a+b}[B(a+c)+A(b+d)]+Bc+Ad} \quad \text{(S)} \quad (9)$$

$$I_c = \frac{E[a(b+d)+g(a+b)]}{R[(a+c)(b+d)+g(a+b+c+d)]+ab(c+d)+dc(a+b)+g(a+b)(c+d)} \quad \text{(C)} \quad (10)$$

$$I_d = \frac{EA}{\frac{R}{a+b}[B(a+c)+A(b+d)]+Bc+Ad} \quad \text{(S)} \quad (11)$$

$$I_d = \frac{E[b(a+c)+g(a+b)]}{R[(a+c)(b+d)+g(a+b+c+d)]+ab(c+d)+dc(a+b)+g(a+b)(c+d)} \quad \text{(C)} \quad (12)$$

Equation for Currents in Terms of the Battery Voltage E, When R = O

These equations are readily obtained from those given above by making R = O, but are given below for convenience.

$$I_g = \frac{E(A-B)}{Bc+Ad} \quad \text{(S)} \quad (13)$$

$$I_g = \frac{E(bc-ad)}{ab(c+d)+dc(a+b)+g(a+b)(c+d)} \quad \text{(C)} \quad (14)$$

$$I_a = \frac{E}{b}\left[\frac{(A-B)(g+b)+Ad}{Bc+Ad}\right] \quad \text{(S)} \quad (15)$$

$$I_a = \frac{E[c(b+d)+g(c+d)]}{ab(c+d)+dc(a+b)+g(a+b)(c+d)} \quad \text{(C)} \quad (16)$$

$$I_b = \frac{E}{b}\left[\frac{(A-B)g+Ad}{Bc+Ad}\right] \quad \text{(S)} \quad (17)$$

$$I_b = \frac{E[d(a+c)+g(c+d)]}{ab(c+d)+dc(a+b)+g(a+b)(c+d)} \quad \text{(C)} \quad (18)$$

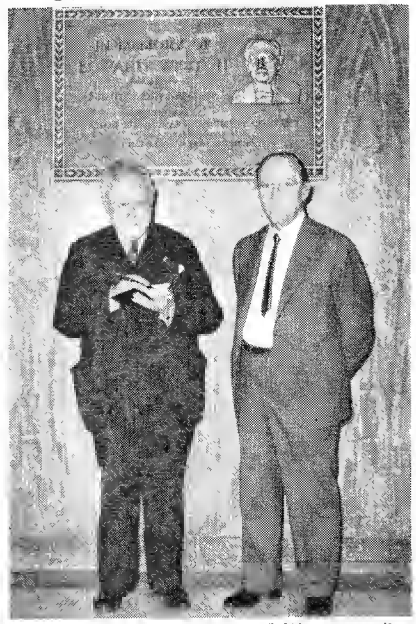
$$I_c = \frac{EB}{Bc+Ad} \quad \text{(S)} \quad (19)$$

$$I_c = \frac{E[a(b+d)+g(a+b)]}{ab(c+d)+dc(a+b)+g(a+b)(c+d)} \quad \text{(C)} \quad (20)$$

(*Continued on page 8*)

Distinguished Italian Visitors at the Weston Laboratories

On August 6, 1947, the Weston Electrical Instrument Corporation was honored by having as visitors to its plant and laboratories the distinguished Italian physicist Professor Giovanni Giorgi, and Professor Enrico Paolini.



Professor Giorgi (left) and Mr. E. F. Weston standing below the memorial plaque to Dr. Weston in the lobby of the new Engineering-Administration Building.

Professor Giorgi, formerly of the University of Rome, Italy, is now head of the Institute of Higher Mathematics of Rome. He is also Superintendent of the Department of Telecommunications of the Italian Government. Professor Paolini is with the University of Milan.

Among Professor Giorgi's many contributions to physics and engineering, the one for which he is best known is the development of the Meter, Kilogram, Second system of units, known as the Giorgi M K S System.

Professor Giorgi found that the electromagnetic, electrostatic, and mechanical systems of units can be combined into one absolute system using the ordinary practical units of physics and engineering, namely the volt, watt, joule, ampere, coulomb, ohm, farad, and henry. This he accomplished by using the meter, kilogram, and second as the fundamental units of length, mass and time, and by assigning 10^{-7} as the numerical value of the permeability of free space. By this means, the three systems of units may be co-ordinated directly without the necessity of first transferring electrical units into abvolts abamperes, etc., and the mechanical units to dynes, ergs, etc., or the application of a factor equal to the speed of light to co-ordinate electrostatic and electromagnetic systems, as is necessary in the classical centimeter gram second (C.G.S.) system used since Maxwell's time.

The Giorgi M K S System was adopted internationally in 1935 by the International Electrotechnical Commission, and many textbooks are now using this system.

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$$I_d=\frac{EA}{Bc+Ad} \qquad \text{(S)} \quad (21)$$

$$I_d=\frac{E[b(a+c)+g(a+b)]}{ab(c+d)+dc(a+b)+g(a+b)(c+d)} \qquad \text{(C)} \quad (22)$$

Equations for Currents in Terms of Battery Current I

$$I_g=I(a+b)\left[\frac{A-B}{B(a+c)+A(b+d)}\right] \qquad \text{(S)} \quad (23)$$

$$I_g=I\left[\frac{bc-ad}{(a+c)(b+d)+g(a+b+c+d)}\right] \qquad \text{(C)} \quad (24)$$

$$I_a=\frac{I(a+b)}{b}\left[\frac{(A-B)(g+b)+Ad}{B(a+c)+A(b+d)}\right] \qquad \text{(S)} \quad (25)$$

$$I_a=I\left[\frac{c(b+d)+g(c+d)}{(a+c)(b+d)+g(a+b+c+d)}\right] \qquad \text{(C)} \quad (26)$$

$$I_b=\frac{I(a+b)}{b}\left[\frac{(A-B)g+Ad}{B(a+c)+A(b+d)}\right] \qquad \text{(S)} \quad (27)$$

$$I_b=I\left[\frac{d(a+c)+g(c+d)}{(a+c)(b+d)+g(a+b+c+d)}\right] \qquad \text{(C)} \quad (28)$$

$$I_c=I(a+b)\left[\frac{B}{B(a+c)+A(b+d)}\right] \qquad \text{(S)} \quad (29)$$

$$I_c=I\left[\frac{a(b+d)+g(a+b)}{(a+c)(b+d)+g(a+b+c+d)}\right] \qquad \text{(C)} \quad (30)$$

$$I_d=I(a+b)\left[\frac{A}{B(a+c)+A(b+d)}\right] \qquad \text{(S)} \quad (31)$$

$$I_d=I\left[\frac{b(a+c)+g(a+b)}{(a+c)(b+d)+g(a+b+c+d)}\right] \qquad \text{(C)} \quad (32)$$

Resistance of Bridge, Looking From Battery

$$\text{Res.}=(a+b)\left[\frac{Bc+Ad}{B(a+c)+A(b+d)}\right]+R \qquad \text{(S)} \quad (33)$$

$$\text{Res.}=\left[\frac{(a+b)cd+g(a+b)(c+d)+ab(c+d)}{(a+c)(b+d)+g(a+b+c+d)}\right]+R \qquad \text{(C)} \quad (34)$$

Resistance of Bridge Looking Out From Galvanometer

This is the resistance which shunts the galvanometer and upon which its damping depends.

$$R_s=\frac{(b+d)ac+bd(a+c)+R(b+d)(a+c)}{(a+b)(c+d)+R(a+b+c+d)} \qquad (35)$$

If $R = O$, or if the bridge is balanced, then:

$$R_s=\frac{(b+d)(a+c)}{a+b+c+d} \qquad (36)$$

NOTE: The quantity

$$ab(c+d)+dc(a+b)$$

appearing in the denominators of the equations may be expressed, if desired, by its equivalent.

$$abcd\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}\right)$$

Sensitivity of Bridge

The galvanometer current ΔIg, produced by a small increase Δd in arm d of the bridge, when initially balanced, i.e., when $bc=ad$, is

$$\Delta I_g=\frac{-Ea\Delta d}{ab(c+d)+dc(a+b)+g(a+b)(c+d)} \qquad (37)$$

NOTE: If d is *decreased* by Δd, the sign becomes $+$.

If the bridge arms are all equal, say equal to arm d, then an increase of Δd in arm d results in a galvanometer current,

$$\Delta I_g=\frac{-E\Delta d}{4d(d+g)} \qquad (38)$$

Conditions for Maximum Wattage in Galvanometer

To find the value of the galvanometer resistance, relative to the resistances of the bridge arms, to produce the maximum wattage in the galvanometer, assuming a constant voltage E across the bridge, i.e., $R = O$.

If the equation for the wattage in the galvanometer circuit, derived from the equation for current is differentiated for a maximum, when the bridge is nearly balanced by varying d, we find that for maximum wattage

$$g=\left[\frac{ab}{a+b}+\frac{dc}{c+d}\right] \qquad (39)$$

If balance is effected by varying, say d, then for near balance condition,

$$g=\frac{b(a+c)}{a+b} \qquad (40)$$

If the bridge arms are all equal, then g is equal to the resistance of *one arm of the bridge.*

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